Adaptive Model Reference Control Based on Takagi-Sugeno Fuzzy Models with Applications to Flexible Joint Manipulators

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The control scheme using fuzzy modeling and Parallel Distributed Compensation (PDC) concept is proposed to provide asymptotic tracking of a reference signal for the flexible joint manipulators with uncertain parameters. From Lyapunov stability analysis and simulation results, the developed control law and adaptive law guarantee the boundedness of all signals in the closed-loop multi-input/multi-output system. In addition, the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

Key Words: Adaptive Control, Fuzzy Control, Flexible Joint Manipulator, Model Reference Control

1. Introduction

Fuzzy logic controllers are generally applicable to plants that are poorly understood in mathematics and where the experienced human operators are available. However, the fuzzy control has not been regarded as a rigorous science due to the lack of the guarantee of the global stability and acceptable performance. To overcome this drawback, since Takagi-Sugeno (TS) fuzzy model (Takagi et al., 1985) which can express a highly nonlinear functional relation in spite of a small number of fuzzy implication rules was proposed, there have been significant research on the stability analysis and systematic design of fuzzy controllers (Tanaka et al., 1992 ; Wang et al., 1996 ; Chen et al., 1996). In their researches, the nonlinear plant is represented by a TS fuzzy model and the controller design is carried out based on the fuzzy model via the so-called Parallel Distributed Compensation (PDC) scheme and Linear matrix inequality based optimization (Hong et al., 2003).

Many researches on the control of flexible joint manipulator have been done such as the model based approaches which include feedback linearization scheme (Khorasani et al., 1990) and invarient manifold scheme (Khorasani et al., 1985a ; Khorasani et al., 1985b) robust control (Spong, 1987) and adaptive control (Lozano et al., 1992 ; Chen et al., 1989). However, although the joint flexibility has demonstrated some potential merits, the difficulty with modelling and controlling such a flexible mechanical system with high performance made most robot designers prefer to manufacture mechanically rigid arms with stiff joints. Hence, in this paper, we will

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tackle the problem of controlling for flexible joint robots via fuzzy modeling and fuzzy model based controller and propose a complete solution to solving the problem of model uncertainty.

In order to deal with the uncertainties of nonlinear systems, in the fuzzy control system literature, a considerable amount of adaptive schemes have been suggested (Chen et al., 1996; Spooner et al., 1996). An adaptive fuzzy system is a fuzzy logic system equipped with an adaptive law. The major advantage of adaptive fuzzy controller over the conventional adaptive controller is that the adaptive fuzzy controller is capable of incorporating linguistic fuzzy information from human operators (Wang et al., 1996; Tsay et al., 1999). Most of them were based on the feedback linearization scheme or indirect adaptive approach in which the approximating ability of the fuzzy system was utilized or an online adaptation scheme was usually used to estimate the unknown structure and parameters of the system and an appropriate controller was then designed to control the plant to satisfy a desired performance (Fischle et al., 1999; Leu et al., 1999). However they all used the original nonlinear plant model as the platform of the stability analysis. If the TS fuzzy modeled plant model can be available, the adaptive scheme based on the obtained TS fuzzy model is much prefer. Hence, in this paper, to control the flexible joint manipulator, we present direct adaptive fuzzy controller based on TS fuzzy model reference approach, in which the desired process response to a command signal is specified by means of a parametrically defined reference model, for MIMO plants with poorly understood dynamics or plants subjected to parameter uncertainties. We utilized TS fuzzy model for flexible joint manipulator configurations with uncertain parameters and PDC as the controllers. The adaptation law for adjusting the parameters in feedback and feedforward gain of PDC controller is designed so that the fuzzy modeled plant output tracks the reference model output.

2. T-S Model Based Control

Consider the continuous-time nonlinear system described by the Takagi-Sugeno fuzzy model. The ith rule of continuous-time TS model is of the following form:

$$R^i: \text{ If } x_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^i \quad \text{Then } \dot{\boldsymbol{x}}(t) = A_i\boldsymbol{x}(t) + B_i\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}^T(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$
$\boldsymbol{u}^T(t) = [u_1(t), u_2(t), \cdots, u_m(t)]$.

Given a pair of input $(\boldsymbol{x}(t), \boldsymbol{u}(t))$, the final output of the fuzzy system is inferred as follows:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{l} w_i(t)\{A_i\boldsymbol{x}(t) + B_i\boldsymbol{u}(t)\}}{\sum_{i=1}^{l} w_i(t)} \tag{2}$$

where $w_i(t) = \prod_{j=1}^{n} M_j^i(x_j(t))$, and $M_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in M_j^i.

In order to design fuzzy controllers to stabilize fuzzy system (Eq. (2)), we utilize the concept of PDC. The PDC controller shares the same fuzzy sets with fuzzy model, Eq. (2) to construct its premise part. That is, the PDC controller is of the following form:

$$R^i: \text{ If } x_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^i \quad \text{then } \boldsymbol{u}(t) = -K_i\boldsymbol{x}(t) \tag{3}$$

where $\boldsymbol{x}^T(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$ and $i = 1, \cdots, l$.

Given a state feedback $\boldsymbol{x}(t)$, the final output of the fuzzy PDC controller, Eq. (3) is inferred as follows:

$$u(t) = -\frac{\sum_{i=1}^{l} w_i(t) K_i\boldsymbol{x}(t)}{\sum_{i=1}^{l} w_i(t)} \tag{4}$$

where $w_i(t) = \prod_{j=1}^{n} M_j^i(x_j(t))$.

By substituting the controller, Eq. (4) into the model, Eq. (2), we can construct the closed-loop fuzzy control system as following:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(t)\, w_j(t)\{A_i - B_iK_j\}\boldsymbol{x}(t)}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(t)\, w_j(t)} \tag{5}$$

A sufficient condition for ensuring the stability of the closed-loop fuzzy system, Eq. (5) is given in Theorem 1, which was derived in the research (Wang et al., 1996).

Theorem 1: The equilibrium of a fuzzy control system, Eq. (5) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$G_{ij}^T P + PG_{ij} = -Q_{ij} \tag{6}$$

for all $i, j=1, 2, \cdots, l$
where $G_{ij}=A_i-B_iK_j$ *and* Q_{ij} *is a positive definite matrix.*

The design problem of model based fuzzy control is to select $K_j (j=1, 2, \cdots, l)$ which satisfy the stability conditions, Eq. (6). In the research (Wang et al., 1996), the common problem was solved efficiently via convex optimization techniques for LMI's (Linear Matrix Inequality) (Hong et al., 2003). However, the fuzzy PDC control, Eq. (4) does not guarantee the stability of system in the presence of parameter uncertainty. Moreover, the design of the control parameters is not possible for the systems whose parameters are unknown. In order to overcome these drawbacks, in this research, an adaptive control scheme is developed for the plant models whose parameters are unknown.

3. Adaptive Model Reference Fuzzy Control

In this section, an adaptive model reference fuzzy control scheme for MIMO TS fuzzy system is developed. Consider again the nonlinear plant represented by the TS model, Eq. (1) or Eq. (2), where state $\boldsymbol{x}\in R^n$ is available for measurement, $A_i\in R^{n\times n}$, $B_i\in R^{n\times q}$ $(i=1, \cdots, l)$ are unknown constant matrices and are controllable. The control objective is to choose the input vector $\boldsymbol{u}\in R^q$ such that all signals in the closed-loop plant are bounded and the plant state $\boldsymbol{x}$ follows the state $\boldsymbol{x}_m\in R^n$ of a reference model specified by the system

$$\dot{\boldsymbol{x}}_m = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\mu_j(\boldsymbol{x})\{(A_m)_{ij}\boldsymbol{x}_m + (B_m)_{ij}\boldsymbol{r}\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\mu_j(\boldsymbol{x})} \tag{7}$$

where $(A_m)_{ij}\in R^{n\times n}$ $(i=1, \cdots, l)$ satisfy the stability condition of fuzzy system given in Theorem 1 and $\boldsymbol{r}\in R^q$ is a bounded reference input vector. The reference model and input $\boldsymbol{r}$ are chosen so that $\boldsymbol{x}_m(t)$ represents a desired trajectory that $\boldsymbol{x}$ has to follow.

If the matrices A_i, B_i were known, we could apply the control law

$$\boldsymbol{u} = \frac{\sum_{j=1}^{l}\mu_j(\boldsymbol{x})(-K_j^*\boldsymbol{x}+L_j^*\boldsymbol{r})}{\sum_{j=1}^{l}\mu_j(\boldsymbol{x})} \tag{8}$$

where $\mu_j(\boldsymbol{x}) = w_j(\boldsymbol{x})$, and obtain the closed-loop plant

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\mu_j(\boldsymbol{x})\{(A_i-B_iK_j^*)\boldsymbol{x}+B_iL_j^*\boldsymbol{r}\}}{\sum_{i=1}^{l}\sum_{j=1}^{l} w_i(\boldsymbol{x})\mu_j(\boldsymbol{x})} \tag{9}$$

Hence, if $K_j^*\in R^{q\times n}$, and $L_j^*\in R^{q\times q}$ are chosen to satisfy the algebraic equations

$$A_i-B_iK_j^*=(A_m)_{ij},\ B_iL_j^*=(B_m)_{ij} \tag{10}$$

then the transfer matrix of the closed-loop plant is the same as that of the reference model and $\boldsymbol{x}(t)\to\boldsymbol{x}_m(t)$ exponentially fast for any bounded reference input signal $\boldsymbol{r}(t)$. However, the design of the control parameters is not possible for the systems whose parameters are unknown. To overcome this drawback, in this research, following controller is developed for the plant models of which parameters are unknown.

Let us assume that K_j^*, L_j^* in Eq. (10) exist, i.e., that there is sufficient structural flexibility to meet the control objective, and propose the control law

$$\boldsymbol{u} = \frac{\sum_{j=1}^{l}\mu_j(\boldsymbol{x})(-K_j(t)\boldsymbol{x}+L_j(t)\boldsymbol{r})}{\sum_{j=1}^{l}\mu_j(\boldsymbol{x})} \tag{11}$$

where, $K_j(t)$, $L_j(t)$ are the estimates of K_j^*, L_j^*, respectively, to be generated by an appropriate adaptive law.

By adding and subtracting the desired input term, namely,

$$\sum_{j=1}^{l}\mu_j(\boldsymbol{x})\{-B_i(K_j^*\boldsymbol{x}-L_j^*\boldsymbol{r})\}/\sum_{j=1}^{l}\mu_j(\boldsymbol{x})$$

in the plant equation and using Eq. (10), we obtain

$$\dot{x}=\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}x+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(B_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}r+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)B_i(K_j^*x-L_j^*r+u)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)} \quad (12)$$

Furthermore, by adding and subtracting the estimated input term multiplied by $\sum_{i=1}^{l}w_iB_i/\sum_{i=1}^{l}w_i$, that is,

$$\frac{\sum_{i=1}^{l}w_iB_i}{\sum_{i=1}^{l}w_i}\left\{\frac{\sum_{j=1}^{l}\mu_j(x)\{(K_j(t)x-L_j(t)r)\}}{\sum_{j=1}^{N}\mu_j(x)}+u\right\}$$

in the reference model (Eq. (7)), we obtain

$$\dot{x}_m=\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}x_m+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(B_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}r+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)B_i(K_j(t)x-L_j(t)r+u)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)} \quad (13)$$

By using Eq. (12) and Eq. (13), we can express the equation of the tracking error defined as $e(t)\triangleq x(t)-x_m(t)$, i.e.,

$$\dot{e}=\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}e+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)B_i(-\tilde{K}_jx+\tilde{L}_jr)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)} \quad (14)$$

where $\tilde{K}_j=K_j(t)-K_j^*$ and $\tilde{L}_j=L_j(t)-L_j^*$.

In the dynamic equation, Eq. (14) of tracking error, B_i is unknown. We assume that L_j^* is either positive definite or negative definite and define $\Gamma_j^{-1}=L_j^*\,sgn(l_j)$, where $l_j=1$ if L_j^* is positive definite and $l_j=-1$ if L_j^* is negative definite. Then $B_i=(B_m)_{ij}L_j^{*-1}$ and Eq. (14) becomes

$$\dot{e}=\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(A_m)_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}e+\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)(B_m)_{ij}L_j^{*-1}(-\tilde{K}_jx+\tilde{L}_jr)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)} \quad (15)$$

Now, by using the tracking error dynamics, Eq. (15), we derive the adaptive law for updating the desired control parameters K_j^*, L_j^* so that the closed-loop plant model, Eq. (12) follows the reference model, Eq. (7). We assume that the adaptive law has the general structure

$$\dot{K}(t)=F_j(x,\ x_m,\ e,\ r)$$
$$\dot{L}=G_j(x,\ x_m,\ e,\ r) \quad (16)$$

where F_j and G_j ($i=1,\ \cdots,\ l$) are functions of known signals that are to be chosen so that the equilibrium

$$K_{je}=K_j^*,\ L_{je}=L_j^*,\ e_e=0 \quad (17)$$

of Eq. (15), Eq. (16) have some desired stability properties.

We propose the following Lyapunov function candidate

$$V(e,\ \tilde{K}_j,\ \tilde{L}_j)=e^TPe+\sum_{j=1}^{l}tr(\tilde{K}_j^T\Gamma_j\tilde{K}_j+\tilde{L}_j^T\Gamma_j\tilde{L}_j) \quad (18)$$

where $P=P^T>0$ is a common positive definite matrix of the Lyapunov equations $(A_m)_{ij}^TP+P(A_m)_{ij}<-Q_{ij}$ for $Q_{ij}=Q_{ij}^T>0$ $(i,j=1,\ \cdots,\ l)$ all whose existence is guaranteed by the stability assumption for A_m. Then, after some straightforward mathematical manipulations with the following properties of trace,

(i) $tr(AB)=tr(BA)$
(ii) $tr(A+B)=tr(A)+tr(B)$ for any $A,\ B\in R^{n\times n}$
(iii) $tr(yx^T)=x^Ty$ for any $x,\ y\in R^{n\times 1}$

we obtain the time derivative $\dot{V}$ of V along the trajectory of Eq. (15) and Eq. (16) as

$$\dot{V}=-e^T\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)Q_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}e$$
$$+2tr\left[-\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)\tilde{K}_j^T\Gamma_j(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,x^T+\sum_{j=1}^{l}\tilde{K}_j^T\Gamma_j\dot{\tilde{K}}_j\right] \quad (19)$$
$$+2tr\left[\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)\tilde{L}_j^T\Gamma_j(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,r^T+\sum_{j=1}^{l}\tilde{L}_j^T\Gamma_j\dot{\tilde{L}}_j\right]$$

In the last two terms of Eq. (19), if we let

$$\sum_{j=1}^{l}\tilde{K}_j^T\Gamma_j\dot{\tilde{K}}_j=\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)\tilde{K}_j^T\Gamma_j(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,x^T \quad (20)$$

$$\sum_{j=1}^{l}\tilde{L}_j^T\Gamma_j\dot{\tilde{L}}_j=-\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)\tilde{L}_j^T\Gamma_j(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,r^T \quad (21)$$

we can make $\dot{V}$ to be negative, i.e.,

$$\dot{V}=-e^T\frac{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)Q_{ij}}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}e\le 0 \quad (22)$$

Hence, the obvious choice for adaptive law to make $\dot{V}$ negative is

$$\dot{\tilde{K}}_j=\dot{K}_j(t)=\frac{\sum_{i=1}^{l}w_i(x)\mu_j(x)(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,x^T \quad (23a)$$
$$=\left\{\frac{\sum_{i=1}^{l}w_i(B_m)_{ij}^T}{\sum_{i=1}^{l}w_i}\right\}\left\{\frac{\mu_j}{\sum_{j=1}^{l}\mu_j}\right\}sgn(l_j)Pe\,x^T$$

$$\dot{\tilde{L}}_j=\dot{L}_j(t)=\frac{\sum_{i=1}^{l}w_i(x)\mu_j(x)(B_m)_{ij}^T sgn(l_j)}{\sum_{i=1}^{l}\sum_{j=1}^{l}w_i(x)\mu_j(x)}Pe\,r^T \quad (23b)$$
$$=\left\{\frac{\sum_{i=1}^{l}w_i(B_m)_{ij}^T}{\sum_{i=1}^{l}w_i}\right\}\left\{\frac{\mu_j}{\sum_{j=1}^{l}\mu_j}\right\}sgn(l_j)Pe\,r^T$$

Theorem 2: Consider the plant model, Eq. (2) and the reference model, Eq. (7) with the control law, Eq. (11) and adaptive law, Eq. (23). Assume that the reference input $\boldsymbol{r}$ and the state $\boldsymbol{x}_m$ of the reference model are uniformly bounded. Then the control law, Eq. (11) and the adaptive law, Eq. (23) guarantee that

(i) $K(t)$, $L(t)$, $e(x)$ are bounded
(ii) $e(t)\rightarrow 0$ as $t\rightarrow\infty$

Proof.

From Eq. (18) and Eq. (22), it directly follows that V is a Lyapunov function for the system, (Eq. (15), Eq. (16)), which implies that the equilibrium given by Eq. (17) is uniformly stable, which, in turn, implies that the trajectory $\tilde{K}(t)$, $\tilde{L}(t)$, $e(t)$ are bounded for all $t>0$. Because $e=x-x_m$ and $x_m\in\mathcal{L}_\infty$, we have that $x=\mathcal{L}_\infty$. From Eq. (11) and $r\in\mathcal{L}_\infty$, we also have that $u\in\mathcal{L}_\infty$; therefore, all signals in the closed-loop are bounded.

Now, let us show that $e\in\mathcal{L}_\infty$. From Eq. (18) and Eq. (22), we conclude that because V is bounded from below and is nonincreasing with time, it has a limit, i.e.,

$$\lim_{t\to\infty}V(e(t),\tilde{K}_j(t),\tilde{L}_j(t))=V_\infty<\infty \quad (24)$$

From Eq. (22) and Eq. (24), it follows that

$$\int_0^\infty e^T\left[\frac{\sum_{i,j=1}^{l}w_i\mu_jQ_{ij}}{\sum_{i,j=1}^{l}w_i\mu_j}\right]ed\tau=-\int_0^\infty\dot{V}d\tau=(V_0-V_\infty) \quad (25)$$

where

$$V_0=V(e(0),\tilde{K}_j(0),\tilde{L}_j(0))$$

On the other hand, from $0\le w_i\le 1$, $0\le\mu_j\le 1$, and

$$\lambda_{min}(Q_{ij})\|e\|^2\le e^TQ_{ij}e\le\lambda_{max}(Q_{ij})\|e\|^2$$

we have

$$\{\lambda_{min}(Q_{ij})\}_{min}\|e\|^2\le e^T\left[\frac{\sum_{i,j=1}^{l}w_i\mu_jQ_{ij}}{\sum_{i,j=1}^{l}w_i\mu_j}\right]e\le\{\lambda_{max}(Q_{ij})\}_{max}\|e\|^2 \quad (26)$$

where

$$\{\lambda_{min}(Q_{ij})\}_{min}=\min\{\lambda_{min}(Q_{11}),\cdots,\lambda_{min}(Q_{ll})\}$$
$$\{\lambda_{max}(Q_{ij})\}_{max}=\max\{\lambda_{max}(Q_{11}),\cdots,\lambda_{max}(Q_{ll})\}$$

After inserting Eq. (26) into Eq. (25), and straightforward manipulation, we have

$$\frac{(V_0-V_\infty)}{\{\lambda_{\min}(Q_{ij})\}_{\min}}\le\int_0^\infty \| e \|^2 d\tau \le \frac{(V_0-V_\infty)}{\{\lambda_{\max}(Q_{ij})\}_{\max}}$$

which implies that $e=\mathscr{L}_2$. Because e, $\tilde{K}_j$, $\tilde{L}_j$, $r\in\mathscr{L}_\infty$, it follows from Eq. (15) that $\dot{e}\in\mathscr{L}_\infty$, which, together with $e\in\mathscr{L}_2$, implies that $e(t)\to 0$ as $t\to\infty$.

4. Numerical Examples

In this section, the validity and effectiveness of the proposed controller are examined through the simulation of tracking control for a flexible joint manipulator.

The control objective is to follow a given trajectory $q_d(t)$ and to produce a torque vector u such that the trajectory error approaches 0 as $t\to\infty$. In the simulation, we examine the effects of parametric variation on behaviors of the closed-loop systems with the proposed TS model based adaptive control scheme.

In order to apply the suggested Adaptive Fuzzy Control (AFC), we need a TS fuzzy model representation of the manipulator.

After the T-S fuzzy model was proposed there have been efforts to construct an efficient T-S fuzzy model for a given nonlinear system. If the T-S fuzzy model is not exactly modeling the nonlinear system, the designed controller may not be able to guarantee the control performance and the stability of the closed loop control system.

To develop a systematic procedure, a T-S fuzzy modeling method, exact T-S fuzzy modeling has recently been developed. The basic idea of exact T-S fuzzy modeling for nonlinear systems has first been discussed in the work (Kawamoto, 1996). Here, the word "exact" means that the defuzzified output of the constructed T-S fuzzy model is mathematically identical to that of the original nonlinear system.

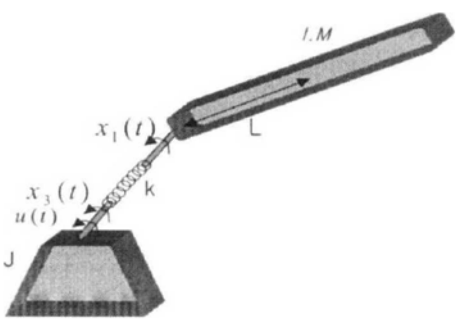

Fig. 1 Flexible joint manipulator configuration

Consider the single link flexible joint manipulator shown in Fig. 1 whose dynamics can be written as

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=-\frac{MgL}{I}\sin x_1-\frac{k}{I}(x_1-x_3)\\ \dot{x}_3&=x_l\\ \dot{x}_4&=\frac{K}{J}(x_1-x_3)+\frac{1}{J}u\end{aligned}\tag{27}$$

where, I, J are, respectively, the link and the rotor inertia moments, M is the link mass, k is the joint elastic constant, L is the distance from the axis of the rotation to the link center of mass and g is the gravitational acceleration respectively. $x_1(t)$ and $x_3(t)$ denote the link angular variable and the actuator shaft angle respectively.

The system, Eq. (27) has a nonlinear term, $\sin(x_1(t))$. If this nonlinear term can be represented as weighted linear sums of some linear functions, then the TS fuzzy model of Eq. (27) can be constructed. For this purpose, we first need the following theorem.

Theorem 3: Consider the following nonlinear term:

$$f_n=x_1x_2\cdots x_n,\ \text{where}\ x_i\in[\Omega_1^i\ \Omega_2^i]$$

It can be exactly represented by a linear weighted sum of the form

$$f_n=\left(\sum_{i_2,i_3,\cdots,i_n=1}^{2}\mu_{i_2i_3\cdots i_n}\cdot g_{i_2i_3\cdots i_n}\right)x_1\tag{28}$$

where $g_{i_2i_3\cdots i_n}=\prod_{j=2}^{n}\Omega_{i_j}^j$, $\mu_{i_2i_3\cdots i_n}=\prod_{j=2}^{n}\Gamma_{i_j}^j$ in which $\Gamma_{i_j}^j$ is positive semi-definite for all $x_j\in[\Omega_1\ \Omega_2]$, defined as follows.

$$\Gamma_1^j=\frac{-x_j+\Omega_2^j}{\Omega_2^j-\Omega_1^j},\ \Gamma_2^j=\frac{x_j-\Omega_2^j}{\Omega_2^j-\Omega_1^j}\tag{29}$$

Proof.

Theorem 3 can be proved using inductive reasoning.

If $n=1$, then Theorem 3 is obviously true. When $n=2$, the nonlinear equation is $f_2=x_1x_2$, which can be represented as the weighted sum of linear functions of x_1 as follows.

$$f_2=\left(\sum_{i_2=1}^{2}\mu_{i_2}g_{i_2}\right)x_1=x_2x_1 \tag{30}$$

where

$$g_1=\Gamma_1^2,\ g_2=\Gamma_2^2,\ \mu_1=\frac{-x_2+\Omega_2^2}{\Omega_2^2-\Omega_1^2},\ \mu_2=\frac{x_2+\Omega_2^2}{\Omega_2^2-\Omega_1^2}$$

Assuming that Theorem 3 holds when $n=k$. then the nonlinear function $f_{k+1}=x_1x_2\cdots x_{k+1}$ can be represented by a weighted linear sum of linear functions of x_1 in the following form.

$$\begin{aligned} f_{k+1}&=\left(\left(\sum_{i_2,i_3,\cdots,i_k=1}^{2}\mu_{i_2i_3\cdots i_k}g_{i_2i_3\cdots i_k}\right)(\Gamma_1^{k+1}Q_1^{k+1}+\Gamma_2^{k+1}\Omega_2^{k+1})\right)x_1\\ &=\left(\sum_{i_2,i_3,\cdots,i_k=1}^{2}\mu_{i_2i_3\cdots i_k1}g_{i_2i_3\cdots i_k1}+\mu_{i_2i_3\cdots i_k2}g_{i_2i_3\cdots i_k2}\right)x_1 \\ &=\left(\sum_{i_2,i_3,\cdots,i_{k+1}=1}^{2}\mu_{i_2i_3\cdots i_{k+1}}g_{i_2i_3\cdots i_{k+1}}\right)x_1\end{aligned} \tag{31}$$

Hence, Theorem 3 holds for all n.

Corollary 1: Assume $x(t)\in[\Omega_1\ \Omega_2]$. The nonlinear term

$$f(x(t))=\sin(x(t)) \tag{32}$$

can be represented by a linear weighted sum of linear functions of the form

$$f(x(t))=\left(\sum_{i=1}^{2}\mu_ig_i(x(t))\right)x(t) \tag{33}$$

where

$g_1(x(t))=1$, $g_2(x(t))=\alpha$ and

$\mu_1=\Gamma_1$, $\mu_2=\Gamma_2$,

$\Gamma_1=\dfrac{\sin(x(t))-\alpha x(t)}{(1-\alpha)x(t)}$, $\Gamma_2=\dfrac{x(t)-\sin(x(t))}{(1-\alpha)x(t)}$

for $x(t)\neq 0$

$\Gamma_1=1$, $\Gamma_2=0$ for $x(t)=0$ and

$\alpha=\sin^{-1}(\max(\Omega_1,\Omega_2))$

proof.

It follows directly from Theorem 3.

Using Corollary 1, an exact TS fuzzy model of Eq. (27) can be represented as follows.

Plant rules:

Rule 1: IF $x_1(t)$ is about Ω_1 THEN

$$\dot{x}(t)=A_1x(t)+B_1u(t)$$

Rule 2: IF $x_1(t)$ is about Ω_2 THEN

$$\dot{x}(t)=A_2x(t)+B_2u(t) \tag{34}$$

where

$$A_1=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{Mgl}{I}-\frac{k}{I} & 0 & \frac{k}{I} & 0\\ 0 & 0 & 0 & 1\\ \frac{k}{J} & 0 & \frac{-k}{J} & 0\end{bmatrix},\ A_2=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{\alpha Mgl}{I}-\frac{k}{I} & 0 & \frac{k}{I} & 0\\ 0 & 0 & 0 & 1\\ \frac{k}{J} & 0 & -\frac{k}{J} & 0\end{bmatrix} \tag{35}$$

$$B_1=B_2=\begin{bmatrix}0\\0\\0\\ \frac{1}{J}\end{bmatrix}$$

and the membership functions '*about* Ω_1' and '*about* Ω_2' are, respectively,

$$\begin{aligned}\Gamma_1(x_1)&=\frac{\sin(x_1(t))-\alpha x_1(t)}{(1-\alpha)x_1(t)}\\ \Gamma_2(x_1)&=\frac{x_1(t)-\sin(x_1(t))}{(1-\alpha)x_1(t)}\\ &\text{for } x_1(t)\neq 0\\ \Gamma_1&=1,\ \Gamma_2=0,\ \text{for } x_1(t)=0\end{aligned} \tag{36}$$

where $\alpha=\sin^{-1}(\max(\Omega_1,\Omega_2))$ and Γ_i is positive definite for all $x_1(t)\in[\Omega_1\ \Omega_2]$. In the simulation, $[\Omega_1\ \Omega_2]$ was chosen as $[-2.85\ 2.85]$.

Although the exact fuzzy model of the flexible joint manipulator does not have any modeling uncertainties since the deffuzzified output of the TS fuzzy model is exactly same to that of original nonlinear flexible joint manipulator Eq. (27), the exact modeling scheme may have some demerit. If the nonlinearities in the system model have very complicated form or the number of them is very large, the methodology presented in Theorem 3 can not be applied easily.

An alternative TS fuzzy modeling technique, the linearization method is often utilized to construct a T-S fuzzy model for a nonlinear system. The linearization based TS fuzzy modeling technique is the most popular as it is simple and the consequent rule base becomes intuitive although the modeling error inevitably exists.

By applying the Lyapunov linearization method (Slotine et al., 1991) at operating points $x_1=-\pi$, 0, π, we obtain the TS fuzzy model for the robot manipulator as followings.

Rule 1 : IF x_1 is about $-\pi$ THEN $\dot{x}=A_1x+B_1u$
Rule 2 : IF x_1 is about 0 THEN $\dot{x}=A_2x+B_2u$ (37)
Rule 3 : IF x_1 is about π THEN $\dot{x}=A_3x+B_3u$

$$A_1=A_3=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{Mgl}{I}-\frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix},\ A_2=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{Mgl}{I}-\frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix} \quad (38)$$

$$B_1=B_2=B_3=\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

The whole state space formed by state vector of the original nonlinear equations is partitioned into three different fuzzy subspaces whose center is located at the center of corresponding membership functions (MF) shown in Fig. 2.

In order to apply the proposed adaptive fuzzy control scheme, the reference model for the plant state $\boldsymbol{x}$ to follow should be specified. In this simulation, the closed-loop eigenvalues for each subsystem are chosen to be the same, which in turn make the reference model for each fuzzy subspace to be the same and linear one as following :

$$\dot{\boldsymbol{x}}_m=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -10 & -10 & -5 \end{bmatrix}\boldsymbol{x}_m+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\boldsymbol{r} \quad (39)$$

The PDC controller shares the same fuzzy sets with fuzzy model to construct its premise part. That is, the PDC controller is of the following form :

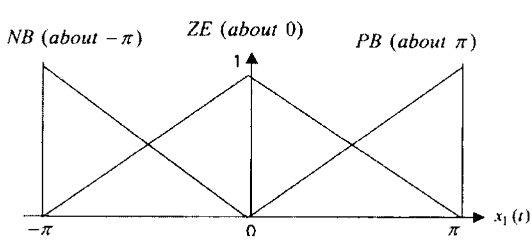

Fig. 2 Membership functions

R^i : If x_1 is MF_i
then $\boldsymbol{u}(t)=-K_i[x_1x_2x_3x_4]^T+L_ir(t)$ (40)

The feedback control gains K_i and L_i of each fuzzy state feedback controller are updated by adaptive law so that the closed-loop plant follows the reference model, Eq. (39).

Now by using Eq. (23), we derive the adaptive law for updating the elements of K_j and L_j so that the closed-loop plant follows the reference model.

$$K_j(t)=\left\{\frac{\mu_j}{\sum_{j=1}^{2}\mu_j}\right\} sgn(l_j)\,B_m^T P e\,\boldsymbol{x}^T$$
$$L_j(t)=\left\{\frac{\mu_j}{\sum_{j=1}^{2}\mu_j}\right\} sgn(l_j)\,B_m^T P e\,\boldsymbol{r}^T \quad (41)$$

where $B_m^T=[0\ 0\ 0\ 1]$.

The parameters of nominal plant model used in this simulation are as follows.

$$I=0.03\ \text{Kg}\cdot\text{m}^2,\ L=1\ \text{m},\ k=31\ \text{N.m}$$
$$J=0.004\ \text{Kg}\cdot\text{m}^2,\ \text{and}\ g=9.8\ \text{m/s}^2 \quad (42)$$

To test the adaptation abilities of the proposed scheme, the mass of link is varied with time as $M(t)=0.2687+0.15\sin 3\pi t$ and k is not known exactly. The initial value for state x_1 is assumed as $x_1=\frac{\pi}{6}$ and the initial parameter of k for the adaptation, $k(0)=28$.

The designed adaptive fuzzy controller was applied to the original nonlinear model of the flexible joint maipulator, Eq. (27) in the simulation. Figs. 3~5 show the simulation results of regulation of joint angle with exact fuzzy model (EFM) and linearization based fuzzy model (LFM). From these figures, It is shown that the regulation problem can be solved under parametric uncertainties. Figs. 6~8 show the tracking control results with both EFM and LFM. In both cases, the tracking can be accomplished successfully. The response characteristics of EFM based control such as response time is better than that of LFM based control. This is due to the fuzzy modeling ability of LFM. If more fuzzy rules, that is, linearization at more operating points can be possible, the difference between both the models can be reduced.

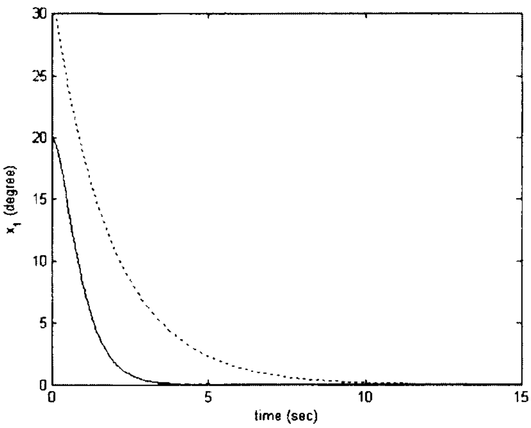

Fig. 3 Regulation using EFM
(solid : x1, dotted : reference)

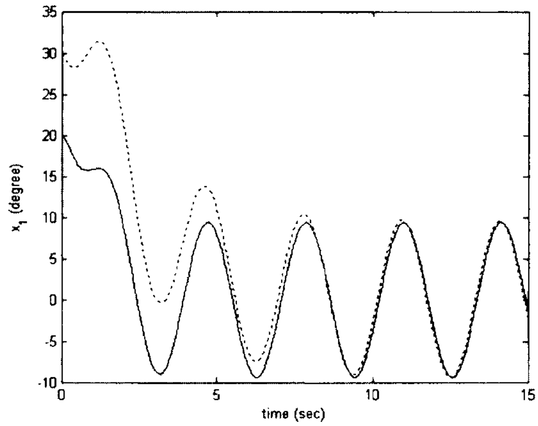

Fig. 6 Tracking using EFM
(solid : x1, dotted : reference)

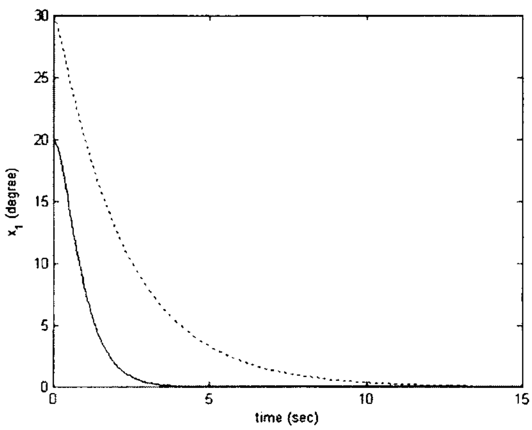

Fig. 4 Regulation using LFM
(solid : x1, dotted : reference)

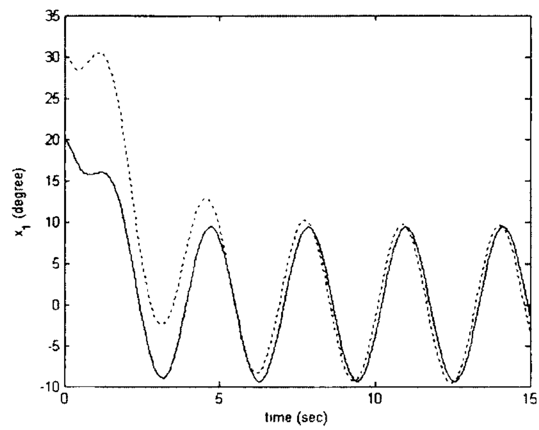

Fig. 7 Tracking using LFM
(solid : x1, dotted : reference)

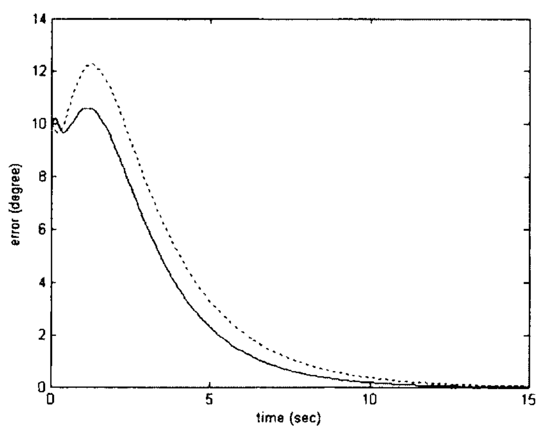

Fig. 5 Regulation error
(solid : EFM, dotted : LFM)

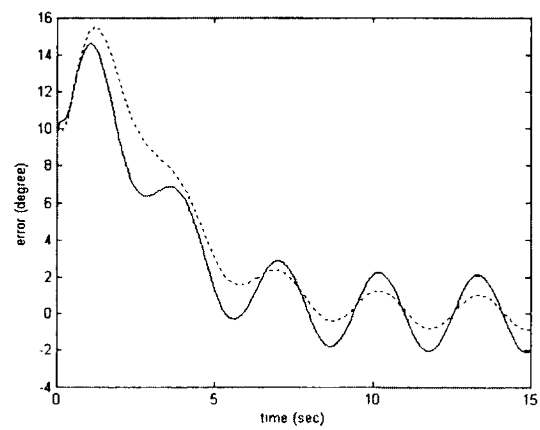

Fig. 8 Tracking error EFM
(solid : LFM, dotted : EFM)

5. Conclusions

In this paper, we have used exact fuzzy modeling method and linearization based modeling method to represent the flexible joint manipulator and the adaptation law adjusts the controller parameters on-line so that the plant output tracks the reference model output. The developed adaptive law guarantees the boundedness of all signals in the closed-loop system and ensures that the plant state tracks the state of the reference model asymptotically with time for any bounded reference input signal.

References

Chen, B. S., Lee, C. H. and Chang, Y. C., 1996, "Tracking Design of Uncertain Nonlinear SISO Systems: Adaptive Fuzzy Approach," *IEEE Tans. Fuzzy Syst.*, Vol. 4, No. 1, pp. 32~43.

Chen, K. P. and Fu, L. C., 1989, "Nonlinear Adaptive Motion Control for a Manipulator with Flexible Joints," *Proc. 1989 IEEE Int. Conf. on Robotics and Automation*, Phoenix, AZ, pp. 1201~1207.

Fischle, K. and Schroder, D., 1999, "An Improved Stable Adaptive Fuzzy Control Method," *IEEE Tans. Fuzzy Syst.*, Vol. 7, No. 1, pp. 27~40.

Hong, S. K. and Nam, Y. S., 2003, "An LMI-Based Fuzzy State Feedback Control with Multi Objectives," *Trans. of KSME*, Vol. 17, No. 1, pp. 105~113.

Kawamoto, S., 1996, "Nonlinear Control and Rigorous Stability Analysis Based on Fuzzy System for Inverted Pendulum," *Proc. IEEE Int. Conf. Fuzzy Systems*, New Orleans, Vol. 2, pp. 1427~1432.

Khorasani, K. and Spong, M. W., 1985, "Invariant Manifolds and Their Application to Robot Manipulators with Flexible Joints," *IEEE Int. Conf. of Robotics and Automation*, St. Louis, MO, pp. 425~428.

Khorasani, K. and Kokotovic, P. V., 1985, "Feedback Linearization of a Flexible Manipulator Near its Rigid Body Manifold," *Systems and Control Letters*, Vol. 6, pp. 187~192.

Khorasani, K., 1990, "Nonlinear Feedback Control of Flexible Joint Manipulator: A Single Link Case Study," *IEEE Trans. Automatic Control*, Vol. 35, No. 10, pp. 1145~1149.

Leu, Y. G., Wang, W. Y. and Lee, T. T., 1999, "Robust Adaptive Fuzzy-Neural Controllers for Uncertain Nonlinear Systems," *IEEE Tans. Robot. and Automat.*, Vol. 15, No. 5, pp. 805~817.

Lozano, R. and Brogliato, B., 1992, "Adaptive Control of Robot Manipulators with Flexible Joints," *IEEE Trans. Automatic Control*, AC-37, pp. 174~181.

Slotine, J. E. and Li, W., 1991, *Applied Nonlinear Control*, Prentice Hall International Editions, New Jersey.

Spong, M. W., 1987, "Modeling and Control of Elastic Joint Manipulators," *ASME Journal of Dynamic Systems, Measurement and Control*, Vol. 109, pp. 310~319.

Spooner, J. T. and Passino, K. M., 1996, "Stable Adaptive Control Using Fuzzy Systems and Neural Networks," *IEEE Tans. Fuzzy Syst.*, Vol. 4, No. 3, pp. 339~359.

Takagi, T. and Sugeno, M., 1985, "Fuzzy Identification of Systems and its Applications to Modelling and Control," *IEEE Trans. Syst., Man, Cybern.*, Vol. 15, No. 1, pp. 116~132.

Tanaka, K. and Sugeno, M., 1992, "Stability Analysis and Design of Fuzzy Control Systems," *Fuzzy Sets and Syst.*, Vol. 45, No. 2, pp. 135~156.

Tsay, D. L., Chung, H. Y. and Lee, C. J., 1999, "The Adaptive Control of Nonlinear Systems Using the Sugeno-Type of Fuzzy Logic," *IEEE Tans. Fuzzy Syst.*, Vol. 7, No. 2, pp. 225~229.

Wang, H. O., Tanaka, K. and Griffin, M. F., 1996, "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues," *IEEE Tans. Fuzzy Syst.*, Vol. 4, No. 1, pp. 14~23.

Wang, L. X., 1996, "Stable Adaptive Fuzzy Controllers with Application to Inverted Pendulum Tracking," *IEEE Tans. Fuzzy Syst.*, Vol. 26, No. 5, pp. 677~691.